Social Choice Violations in Rank Sum Scoring: 
A Formalization of Conditions and Corrective Probability Computations

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Abstract: Rank sum scoring is a popular manner by which to obtain group rankings from events that are individual in nature. For example, rank sum scoring is employed in cross country team competition, the Putnam Mathematics Team Competition, snowboard cross team competition, snowboard half-pipe team competition, and snowboard slopestyle team competition. It also forms the basis for a non-parametric statistical test (the Wilcoxon rank-sum test) that serves as an ordinal substitute for the two-sample t-test. Moreover, a close variant of rank sum scoring is used in team rowing competitions, alpine ski competitions, as well as in multiple event crossfit contests. Given its applicability, we formally define rank sum scoring, derive conditions under which the methodology leads to violations of major social choice principles (ranking cycles and violations of “independence from irrelevant groups”), and calculate the probability that an “outcome sequence” leads to a given social choice violation. These probability calculations provide sizable corrections to prior calculations on the topic, and alter our view as to the social choice properties of rank sum scoring. Contrary to prior results on the topic, we find that the two violations considered are not mutually inclusive under rank sum scoring.