Dimension of Splines and Supersmoothness.

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Abstract: Using only standard tools from multivariate calculus, we show that if we continuously glue two smooth functions along a curve with a “corner”, the resulting continuous function must be differentiable at the corner, as if to compensate for the singularity of the curve. Moreover, locally, this property characterizes non-smooth curves. We also generalize this phenomenon to higher order derivatives. Next we investigate supersmoothness of a special class of functions: smooth piecewise polynomials or splines. Splines of fixed polynomial degree and fixed global smoothness form vector spaces, while splines of fixed global smoothness form modules over the ring of polynomials. Finding the dimension of spline spaces is an important open problem in numerical analysis and algebraic geometry. We show how supersmoothness can help.