GRADUATE COURSE OUTLINES
500 and 600 Level

MATHEMATICS DEPARTMENT

WESTERN ILLINOIS UNIVERSITY

Fall 2015

Contents

1

Core Courses

1.1 Math 551: Methods of Classical Analysis (3 s.h.)

Catalog Description

Introduction to complex and multivariable analysis with a significant lean towards applications. Topics include geometry of R^n, differential calculus in R^n, line and surface integrals; conformal mappings, complex integration, Laurent series, calculus of residues; and applications.

Prerequisites: MATH 231 and MATH 311, or equivalents.

Course Outline

1. Multivariable analysis
   (a) Geometry of R^n: dot product, length, angle, and Schwartz's inequality.
   (b) Motion in R^3: velocity and acceleration, curvature and torsion.
   (c) Differential calculus in R^n: directional derivatives, Jacobean matrix, chain rule, and higher order partial derivatives.
   (d) Del operator: gradient, divergence, curl, and Laplacian.
   (e) Conservative vector fields and independence of path.
   (f) Streamlines and equipotentials.
   (g) Green's Theorem, Stokes' Theorem, and Divergence Theorem.
   (h) Applications to fluid dynamics.
2. Complex Analysis
   (a) Complex numbers
   (b) Analytic functions and conformal mappings
   (c) Complex line integral
   (d) Cauchy's theorem and Cauchy integral formula
   (e) Laurent series
   (f) Calculus of residues and contour integration
   (g) Applications to fluid dynamics
3. Additional applications
   (a) Steady state heat flow

(b) Electromagnetism

(c) Fourier and Laplace transforms

(d) Z-transforms

References

1. Gamelin, T.W. (2001), *Complex Analysis*, Springer-Verlag.
2. Hubbard, J. (2007), *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach*, 3rd edition.
3. Kaplan, W. (2002), *Advanced Calculus*, 5th edition.

Last revised: Spring 2013

1.2 Math 552: Scientific computing (3 s.h.)

Catalog Description

Design, analysis, and MATLAB or Mathematica implementation of algorithms for solving problems of continuous mathematics involving linear and nonlinear systems of equations, interpolation and approximation, numerical differentiation and integration, and ordinary differential equations with a significant lean toward applications.

Prerequisites: MATH 311 and MATH 333, or equivalents.

Course Outline

1. Introduction to algorithms and computer Arithmetic
2. Solution of Nonlinear Equations
   - **(a)** Bisection Method
   - **(b)** Fixed point methods, Newton's Method, Secant Method
   - **(c)** Error Analysis
3. Interpolation and Approximation
   - **(a)** Polynomial Interpolation (Lagrange, Hermite Polynomials)
   - **(b)** Interpolation using Spline Functions
   - **(c)** Least Squares Approximation
4. Numerical Differentiation and Integration
   - **(a)** Trapezoidal and Simpson Rules
   - **(b)** Gaussian Numerical Integration
   - **(c)** Numerical Differentiation
5. Solutions of Linear Systems
   - **(a)** Gaussian Elimination
   - **(b)** LU factorization
   - **(c)** Gauss-Seidel and Jocobi's methods*
6. Solution of Ordinary Differential Equations
   - **(a)** Euler's Method
   - **(b)** Convergence Analysis of Euler's Method
   - **(c)** Runge-Kutta Methods*

* Optional topics

References

1. Scientific Computing with Matlab and Octave by Quarteroni, Saleri & Gervasio.
2. Numerical Computing with MATLAB by Moler, revised reprint, 2004.
3. Scientific Computing: An Introductory Survey, Michael Heath, McGraw-Hill.

Last revised: Spring 2013

1.3 Stat 553: Applied Statistical Methods (3.s.h.)

Catalog Description

Introduction to probability and statistics with a significant lean toward applications. Topics include probability, probability distributions, Central Limit Theorem, sampling distributions (t, F, Chi-Square), parameter estimation, hypothesis testing, nonparametric statistics, ANOVA, and linear regression.

Prerequisites: MATH 231 and STAT 276, or equivalents.

Course Outline

- Review of Probability and Counting Principles
- Random Variables and Probability Distribution Functions
- Survey of Probability Distributions (Binomial, Normal)
- Sampling Distributions : T, F, Chi-Square
- Point Estimation
- Confidence Interval Estimation for One and Two Samples
- Hypothesis Testing: One and Two Sample Problems
- Categorical Data Analysis: RxC Contingency Tables
- Linear Regression
- Nonparametric Statistics

Optional Topics (Not limited to the following topics)

- Bootstrap Estimation
- ANOVA (Analysis of Variance)
- MANOVA (Multivariate Analysis of Variance)
- ANCOVA (Analysis of Covariance)

References

1. D. Wackerly, W. Mendenhall, R. L. Scheaffer, Mathematical Statistics with Applications, Thomson Brooks/Cole, 2008

Last revised: Spring 2012

1.4 Math 651: Elements of Modern Analysis (3 s.h.)

Catalog Description

A study of elements of modern analysis with a lean toward developing theory. Topics include Lebesgue's theorem on Riemann integrability; metric space compactness, completeness and completions; pointwise and uniform convergence; normed vector spaces; Banach fixed point theorem, Weierstrass approximation theorem; and applications.

Prerequisites: MATH 435 and MATH 551, or equivalents.

Course Outline

1. Riemann Integration: Integrability conditions including almost everywhere continuity and Riemann-Lebesgue theorem
2. Pointwise and Uniform Convergence: Definitions, examples, and applications to continuity, differentiability, and integrability of the limit function. The development will include both sequences and series of functions.
3. Point-set Topology: limits and continuity; compactness including its characterization in metric spaces; connectedness; metric space completeness and completions; uniform continuity; Lipchitz continuity; Banach fixed point theorem; Weierstrass approximation theorem
4. Basic Theory of Normed Vector Spaces: Definitions, basic properties and examples of Banach and Hilbert spaces; characterization of compactness in finite-dimensional normed vector spaces.
5. Applications: Fourier series and its applications; PicardLindelf theorem and its applications to existence and uniqueness of solutions of ordinary differential equations.
6. Additional topics: Equicontinuity and Arzela-Ascoli theorem; Invertible operators and existence of solutions for the Fredholm and Voltera integral equations; Approximations in convex normed spaces and existence of a best approximation; Least squares approximation and regression analysis.

References

1. Davidson, K. R. and Donsig, A. P. (2010), *Real Analysis and Applications: Theory in Practice*, Springer.
2. Pugh, C.C. (2010), *Real Mathematical Analysis*, Springer.
3. Kreyszig, E. (1989), *Introductory Functional Analysis with Applications*, Wiley.

Last revised: Spring 2012

1.5 Math 652: Computational Differential Equations (3 s.h.)

Catalog Description

A study of elements of computational mathematics of differential equations with a lean toward developing the theory and concentration on ordinary differential equations. Topics include multi-step methods, Runge-Kutta and collocation methods for ordinary differential equations, error, stiffness, and stability analysis, theory of finite differences with application to partial differential equations, and direct and iterative methods for sparse linear systems.

Prerequisites: MATH 435 or MATH 551, and MATH 552 or MATH 481.

Course Outline

1. Ordinary Differential Equations
   - One-step and multi-step methods for ODEs with error control.
   - Accuracy, stability, and convergence properties.
   - Modifications of Newton's method for implicit solvers on stiff problems.
2. The Poisson Equation
   - Finite Difference and Finite Element Methods
   - Direct and Iterative methods for sparse linear systems
   - Multigrid techniques and Fast Poisson Solvers.
3. PDEs of Evolution
   - Diffusion equation
   - Hyperbolic equations
   - Method of Lines

References

1. A First Course in the Numerical Analysis of Differential Equations, A. Iserles, Cambridge Texts in Applied Mathematics

Last revised:

1.6 Stat 653: Elements of Statistical Inference (3.s.h.)

Catalog Description

A study of elements of statistical inference with a lean toward developing the theory. Topics include probability theory, random variables, probability distribution functions, limit theorems, estimation, testing, sufficiency, robust statistical methods, bootstrap, and linear models.

Prerequisites: STAT 471 and STAT 553.

Course Outline

- Probability Theory
- Some Probability Distributions: Discrete/Continuous Cases
- Mathematical Expectation, Higher Moments, Variance
- Chebyshev's Inequality, Convergence, Limit Theorems
- Point Estimation Theory
- Properties of Estimators: Unbiasedness, Efficiency, Consistency, The Rao-Cramer Lower Bound
- Confidence Interval Estimation Theory
- Hypothesis Testing Theory: The Neyman-Pearson Lemma, Likelihood Ratio Tests, Type-I and Type-II Error, Power of Tests
- Bayesian Approach to Statistical Inference
- Nonparametric Methods
- Bootstrap Methods

Optional Topics (Not limited to the following topics):

- Categorical Data Analysis
- Regression
- Design and Analysis of Experiments

References

1. R. V. Hogg, J. McKean, A. T. Craig, Introduction to Mathematical Statistics , Pearson, 2012

Last revised: Spring 2012

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Focus Courses

2.1 Math 521: Algebra (3 s.h.)

Catalog Description

An introduction to higher algebra. Topics to be included are groups, homomorphisms, rings and ideals, fields, field extensions, and Galois theory.

Prerequisites: MATH 424 or permission of the instructor.

Course Outline

1. External direct product
2. Fundamental theorem of Finite Abelian groups
3. Rings
   - Integral Domains
   - Ideals
   - Polynomial rings
4. Fields
   - Field extensions
   - Finite fields
   - Geometric constructions

Additional topics

1. Sylow Theorem
2. Galois theory

Remarks: MATH 521 is a focus course. The content is selected at the discretion of the instructor.

References

1. Contemporary abstract algebra by J.A. Gallian
2. Modern algebra an introduction by J. Durbin

Last revised: Currently under review

2.2 Math 533: Complex Variables (3 s.h.)

Catalog Description

Topics to be studied include the topology of complex plane, analytic functions, complex integration, and singularities.

Prerequisites: MATH 435 or permission of the instructor.

Course Outline

Remarks: MATH 533 is a focus course. The content is selected at the discretion of the instructor.

Last revised:

2.3 Math 536: Ordinary Differential Equations (3 s.h.)

Catalog Description

Initial value problems, existence and uniqueness theorems, linear systems, asymptotic behavior of solutions, two dimensional systems

Prerequisites: MATH 333 and 435, or permission of the instructor.

Course Outline

1. Existence and uniqueness results for initial value problems
2. Continuation of solutions and maximal intervals of existence
3. Linear equations and systems
4. Stability theory
5. Sturm-Liouville theory and boundary value problems

Remarks: MATH 536 is a focus course. The content is selected at the discretion of the instructor.

References

1. Nonlinear Systems, by H. Khalil, 3rd Ed.

Last revised: Currently under review

2.4 Math 554: Method of Symmetry in Algebra, Geometry, and Topology (3 s.h.)

Catalog Description

A study of symmetry in algebra, geometry, and topology with a significant lean toward applications. Topics of study include group of Euclidean transformations, symmetries of planar sets, topological classification of compact surfaces, crystallographic patterns and classification of their symmetry groups.

Prerequisite: MATH 424 or permission of the instructor

Course Outline

1. Congruence via Euclidean transformations and equivalence relations.
2. The group of Euclidean transformations.
3. Group actions and orbit spaces.
4. Finite subgroups of the group of Euclidean transformations of the plane $\mathcal{I}(E^2)$.
5. Symmetry groups of subsets of Euclidean plane (space).
6. Classification of symmetries of patterns on a disc.
7. Crystallographic patterns and the action of their groups of symmetries.
8. Topological classification of surfaces
9. Graphs on surfaces, Euler characteristic, and Gauss-Bonnet Theorem.
10. Classification of the symmetry groups of crystallographic patterns.

Remarks: MATH 554 is a focus course. The content is selected at the discretion of the instructor.

References

1. D. L. Johnson, Symmetries, Springer-Verlag, 2001

Last revised:

2.5 Math 560: Advanced Topology (3 s.h.)

Catalog Description

Product and quotient spaces, path–connectedness, local compactness, homotopy, fundamental group, Additional topics may include Baire category, functional spaces, Brouwer fixed point theorem.

Prerequisite: MATH 421 and 461 or permission of the instructor

Course Outline

Remarks: MATH 560 is a focus course. The content is selected at the discretion of the instructor.

References

1. J. Munkres, Topology, Prentice–Hall, 2000

Last revised:

2.6 Math 581: Approximation Theory (3 s.h.)

Catalog Description

The theory behind the numerical algorithms. Remainder theory, convergence theorems, best approximation in various norms, the theory of matrices in numerical analysis including eigenvalue problems.

Prerequisites: MATH 435 and 481, or permission of the instructor

Course Outline

1. Approximations in normed spaces
   - Existence
   - Uniqueness and strict convexity
   - Uniform approximation
   - Least square approximation
2. Polynomial approximations
   - Weierstrass Approximation Theorem
   - Lagrange and Hermite polynomial interpolation
   - Least square polynomial approximation
3. Piecewise polynomial approximation
   - Piecewise linear interpolation
   - Cubic spline interpolation
4. Rational Approximation
   - Best rational approximations in the uniform norm
   - Pade approximations
   - Rational interpolation

Remarks: MATH 581 is a focus course. The content is selected at the discretion of the instructor. An introduction to interpolation and approximations is covered in Math 552. MATH 581 provides a comprehensive discussion to approximation theory.

References

1. M. J. D. Powell, Approximation Theory and Methods, Cambridge University Press, 1981

Last revised: Currently under review.

2.7 Math 583: Nonlinear optimization (3 s.h.)

Catalog Description

Unconstrained optimization; equality constrained optimization; convex optimization; optimality conditions; algorithms and applications using software such as Mathematica.

Prerequisites: MATH 424 and 481, or permission of the instructor

Course Outline

1. Local Unconstrained Optimization
   (a) Optimality conditions
   (b) Algorithms – Descent methods, Newton's method
2. Equality constraints Optimization
   (a) Lagrange Multiplier Theorem
   (b) Algorithms – Newton's method with equality constraints
3. Convex optimization.
   (a) Convex sets and functions
   (b) Convexity conditions
   (c) KKT optimality conditions
   (d) Algorithms – Interior-point methods
4. Applications

Remarks: MATH 583 is a focus course. The content is selected at the discretion of the instructor.

References

1. Bertsekas, D.P. (2004), *Nonlinear Programming*, Athena Scientific.
2. Boyde, S. and Vandenberghe, L. (2004), *Convex optimization*, Cambridge University Press.

Last revised: Spring 2013

2.8 Math 596: Project in Applied Mathematics (3 s.h. (repeatable to 6))

Catalog Description

A project in applied mathematics or statistics, or with a professional institution, which will be presented in a final paper or portfolio, demonstrating entry into an applied mathematics field. Graded S/U. Prerequisites: Permission of the Graduate Committee.

Protocol

Any faculty member responsible for guiding the project serves as the *Project Supervisor*. The student must submit the project outline to the Coordinator of the Graduate Program and receive approval.

Application Requirements

1. Minimum G.P.A. required: 3.0
2. Students may count up to 6 s.h. towards the focus area courses.
3. Application must be submitted before to the end of the semester prior to the proposed project.

Registration Requirements

Complete the project application and submit it with the project supervisor's signature. Signing the application form confirms that the project supervisor will take the necessary steps to complete the project. The department chair signs the application, approves the project, and gives permission for registration.

Student Responsibility

It is the responsibility of the student to complete a project report, which must be approved by the supervisor, and a summary of the project. These must be provided to the Graduates Coordinator. Failure on the part of the student to submit the report or the project summery may result in a grade of "Unsatisfactory", which is equivalent to no credit earned for the project. Students who do not complete the requirements by the end of a term will receive an incomplete ("I"). All requirements must be completed by the sixth week of the term following the project to remove an "I".

Project Supervisor Responsibility

Submit a letter of evaluation of the student's performance and approve of the report.

Graduate Coordinator Responsibility

Evaluate documentation submitted by the student and the project supervisor. Assign a "S/U" grade at the completion of evaluation.

Last revised: Fall 2014

2.9 Math 600: Thesis (3 s.h.)

Catalog Description

The thesis is either expository, historical, critical, or original and must be approved by the student advisory committee. The student must present his/her thesis to the mathematics department faculty in a colloquium. Prerequisites: Permission of the Graduate Committee.

2.10 Math 601: Advanced Project in Applied Mathematics (3 s.h. (repeatable to 6)

Catalog Description

A project in an advanced topic of mathematics or statistics, which will be presented in a final paper or portfolio, demonstrating advanced proficiency in an applied mathematics field. Graded S/U. Prerequisites: Permission of the Graduate Committee.

Protocol

Any faculty member responsible for guiding the project serves as the *Project Supervisor*. The student must submit the project outline to the Coordinator of the Graduate Program and receive approval.

Application Requirements

1. Minimum G.P.A. required: 3.0
2. Must have completed 18 s.h. of graduate studies.
3. Students may count up to 6 s.h. towards the focus area courses.
4. Application must be submitted before to the end of the semester prior to the proposed project.

Registration Requirements

Complete the project application and submit it with the project supervisor's signature. Signing the application form confirms that the project supervisor will take the necessary steps to complete the project. The department chair signs the application, approves the project, and gives permission for registration.

Student Responsibility

It is the responsibility of the student to complete a project report, which must be approved by the supervisor, and a summary of the project. These must be provided to the Graduates Coordinator. Failure on the part of the student to submit the report or the project summery may result in a grade of "Unsatisfactory", which is equivalent to no credit earned for the project. Students who do not complete the requirements by the end of a term will receive an incomplete ("I"). All requirements must be completed by the sixth week of the term following the project to remove an "I".

Project Supervisor Responsibility

Submit a letter of evaluation of the student's performance and approve of the report.

Graduate Coordinator Responsibility

Evaluate documentation submitted by the student and the project supervisor. Assign a "S/U" grade at the completion of evaluation.

Last revised: Fall 2014

2.11 Math 602: Internship in Applied Mathematics (3 s.h.(repeatable to 6))

Catalog Description

Internship in Applied Mathematics - Mathematical work or training conducted at a professional institution, university or government organization, which will be presented in a final paper or portfolio, demonstrating advanced proficiency in an applied mathematics field. Graded S/U.

Prerequisite: Permission of the Graduate Committee.

Designated Personal

A. The chair of the graduate committee coordinates all the internship activities and serve as the Internship Coordinator of the department. All internships must first be approved by the coordinator.

B. Any faculty member responsible for monitoring an internship while it is in progress serves as the Internship Supervisor.

Protocol

The student must prepare a proposal, submit it to the internship coordinator and be granted approval.

Application Requirements

A. Minimum overall G.P.A. required: 3.0

B. Minimum major G.P.A. required: 3.0

C. Students may count toward the major up to 3 s.h. of each MATH 602 internship, for a maximum of 6 s.h.
The number of credit hours will be determined by the department graduate committee after examining the description and the hours of the activities undertaken for the internship.

D. The internship site must have an existing Internship Program within its organization.

E. The application must be submitted in advance of the date of the proposed internship.
SPECIFICALLY, THIS MUST BE DONE PRIOR TO FINAL EXAM PERIOD OF SEMESTER PRIOR TO INTERNSHIP.

Registration Requirements

A. Submit the completed proposal.

B. Submit documentation of the Company's Internship Program.

C. Submit the Documentation from the company regarding the specific internship site and work assigned.

D. Provide the on-site supervisor contact information and offer letter (if available) from the internship program.

Student Internship Responsibility

It is the responsibility of the interning student to complete all the requirements listed below and submit materials to the Department Internship Supervisor.

A. Weekly Log - Include duties performed, experiences and personal insights for each day.

B. Final Internship Report - The student will submit to the Faculty Internship Supervisor a report on the activities undertaken during the internship. This report is to be detailed and relate the activities to the student's degree program. The frequency with which the student should report on internship activities during the internship is at the discretion of the Faculty Internship Supervisor.

C. Letter of Evaluation - Request a letter of evaluation regarding the student's internship performance from the on-site supervisor, to be sent to directly to Department Internship Coordinator or the student's Internship Supervisor.

The weekly log, final internship report, and letter of evaluation must be submitted to the Department Internship Coordinator and to the student's Internship Supervisor before credit can be awarded.

On-Site Supervisor Responsibility

Write a letter of evaluation of the intern's performance during the internship. Mail it directly to Department Internship Coordinator or to the student's Internship Supervisor at the completion of the internship.

Faculty Internship Supervisor Responsibility

Serve as the faculty contact for the internship provider. Evalaute the weekly log and grade the final Report of Internship Activities.

Department Internship Coordinator Responsibility

A. Evaluate the internship documentation and offer letter submitted by the student and approve the internship registration.

B. Assign a grade at the completion of the internship, with the support of the faculty internship supervisor.

Last revised: Spring 2015

2.12 Math 655: Technology and the Secondary School Mathematics Curriculum (3.s.h)

Catalog Description

Strategies for using technology such as calculators, computers, and Internet resources for teaching algebra, geometry, probability, and statistics in the secondary mathematics curriculum, including research on the use of technology for mathematics teaching and learning.

Prerequisites: Graduate Standing and and permission of the instructor.

Course Outline

I Overview of the Technology Resources and Instructional Issues.

- **A** Introduction to the Use of Calculators(Including graphics calculators and programming)
- **B** Introduction to the Use of Computers
  1. Basic principles, menus, and file management.
  2. Basics of electronic spreadsheet programming.
  3. Multimedia software.
- **C** Introduction to Internet Use
  1. Navigation, search, and downloading basics.
  2. Creating a World Wide Web homepage and HTML programming.
  3. Internet site evaluation.
- **D** Introduction to Planning for Instruction with Technology Issues
  1. Publications and research on the use of technology in the mathematics classroom.
  2. Classroom structure.
  3. Teacher tools.

II A Closer Look at Technology and Instructional Issues

- **A** Algebra-Investigating Functions and Relations Using Technology
  Using calculators, computers, the Internet, and planning for instruction.
- **B** Investigating and Discovering Geometry Concepts Using Technology
  Using calculators, computers, the Internet, and planning for instruction.
- **C** Probability and Statistics (Data Analysis) Using Technologies
  Using calculators, computers, the Internet, and planning for instruction.

Note that Part I (Introduction) makes up about 25% of the course. Part II (Closer Look) makes up 75% of the course.

References

1. Technology Standards for All Illinois Teachers (Illinois State Board of Education) http://www.isbe.net/profprep/PDFs/techstandards.pdf
2. National Educational Technology Standards for Teachers from the International Society for Technology in Education http://cnets.iste.org/
3. Principles and Standards for School Mathematics, National Council of Teachers of Mathematics (2000) http://standards.nctm.org/
4. Journals to be used are listed below under Library Resources.

- Mathematics Teaching in the Middle School
- Mathematics Teacher
- Online Journal for School Mathematics (ON-Math)
- Journal of Online Mathematics and its Applications (JOMA)
- Journal for Research in Mathematics Education

Last revised:

2.13 Math 656: Advanced Perspective of Secondary School Mathematics, (3.s.h)

Catalog Description

An advanced study of the mathematics of secondary school curriculum for the purpose of developing deeper connections and representations for all students. Focus is on rigorous conceptual content knowledge, methods of inquiry, and investigative problem-solving. Topics include Algebra, Geometry, and Statistics.

Prerequisites: Graduate standing and permission of the Department Chair.

Course Outline

- Functions, Equations and Solving
- Number sets, systems and structures
- Integers and Polynomials
- Congruence, similarity, and distance
- Area and Volume
- Axiomatics and Euclidian Geometry

References

Mathematics for High School Teachers- An Advanced Perspective By Usiskin, Peressini, Marchisotta, Stanley

Note: Drs Stanley (Berkeley) and Peressini (UIUC) conducted a weeklong workshop using this text on our campus in the summer of 2003 and this course would be built upon similar principles.

2.14 Stat 570: Probability Theory and Stochastic Processes (3.s.h)

Catalog Description

Nature of probability theory, sample space, combinatorial analysis, fluctuations in random events, stochastic independence, random variables, generating functions, Markov chains, and simple time-dependent stochastic processes.

Prerequisites: STAT 471 or equivalent.

Course Outline

- Events and Probability Spaces
- Basic Theorems of Probability
- Random Variables and Expectations
- Limit Theorems
- Moment Generating Functions and Related Transforms
- Common Discrete and Continuous Distributions Markov Chains
- The Poisson Process and Related Processes

Selected Additional Topics:

- Basic Renewal Theory with applications
- Some basic stochastic models

References

1. Ross, S.M. (1989), Introduction to Probability Models, Fourth Edition, Academic Press.