1. [4 Points] The weather on capricious planet Venus changes according to a Markov process. Each day is either “clear” or “cloudy”. If it’s clear today, there is a 40% chance to have clouds tomorrow. If it is cloudy one day, there is a 55% chance to see clear skies the following day.

   a) Set up the transition matrix for this Markov process.

   b) Suppose it was cloudy on Wednesday. What is the probability to have clear skies on Friday?

   c) If the forecast for Tuesday is “60% chance of clear skies” what do you predict for Friday (same week)?

   d) In the long run, what will you experience more often: clear or cloudy? What is the probability to have a cloudy day in the long run?

2. [4 Points] For a group of states it was observed that 60% of the Democratic governors were succeeded by Democrats and 55% of the Republican governors were succeeded by Republicans.

   a) Set up the transition matrix for this process:

   b) Suppose that 70% of governors are Republicans now. What percentage of the governors will be Democrats after two elections?

   c) What percentage of the governors will be Democrats in the long run?
3. [6 Points] Consider three resorts in Florida competing with each other. Tourists who chose one resort this year might prefer another resort for the next year vacation. Suppose the process is described by the following matrix:

\[ A = \begin{bmatrix} .6 & .55 & .2 \\ .19 & .25 & .3 \\ .21 & .2 & .5 \end{bmatrix}. \]

a) Which resort does manage to keep its clients best?

b) Which resort simply doesn’t know how to keep its customers?

c) Find the percentage of people who would switch from resort II to resort I for the following year.

d) Explain the meaning of the percentage .21 (21%) in the matrix.

e) Suppose in 2005 there were 4000 clients in each resort. What would you predict for 2007 (how many will be in each resort)?

f) Which resort would be the most successful in the long run? What percentage of all the tourists will choose that resort in the long run?

4. [3 Points] Describe Markov “process” characterized by the transition matrix:

\[ A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \]
5. [4 Points] Determine which of the following matrices are stochastic:

\[
\begin{bmatrix}
0 & .1 \\
1 & .9 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.22 & .3 & .5 \\
.08 & .4 & .3 \\
.7 & .3 & .4 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-.1 & .3 \\
1.1 & .7 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & .3 & .5 \\
0 & .7 & .3 \\
0 & 0 & .2 \\
\end{bmatrix}
\]

6. [4 Points] Determine which of the following matrices are regular stochastic:

\[
\begin{bmatrix}
.2 & .1 \\
.8 & .9 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.2 & .5 & .5 \\
.1 & .1 & .2 \\
.7 & .3 & .3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & .1 \\
0 & .9 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
.8 & 1 \\
.2 & 0 \\
\end{bmatrix}
\]

7. [3 Points] Suppose a certain football team must win all the remaining 5 games in order to qualify for a bowl game. Assume winning is as likely as loosing. What is the probability that the team will qualify for a bowl game?

8. [3 Points] Suppose there are 30 students in a certain finite math class, 20 of whom are business majors, 8 are computer science majors, and 2 others have some other majors. Suppose 15 students are freshmen and 7 students are non-freshmen and non-business majors.
a) Find the probability that a student in this class is a business major.

b) Find the probability that a student in this class is a freshman.

c) Find the probability that a student in this class is a business major given that he/she is a freshman.

d) Find the probability that a student in this class is a freshman given he/she is a business major.

9. [3 Points] Find

\[ Pr(\text{Cardinals won World Series 2005}|\text{White Sox won World Series 2005}) = \]

10. [3 Points] A TV repair shop uses a two-step diagnostic procedure. Step I locates the problem in a TV with probability .75. Step II (if it is needed) identifies the problem with probability .65.

   a) Show the tree diagram for this process.

   b) Find the probability that the diagnostic fails to identify the problem.

11. [3 Points] You roll a 6-sided die. If you get an even number, you draw a card from the standard deck. Otherwise, you toss a coin.

   a) Show the tree diagram.

   b) What is the probability you end this process with a Queen?

   c) What is the probability you end this process with heads on the coin?