1. Let $H$ be a subgroup of a group $G$, written multiplicatively.
   (a) Define a group action $H \times G \rightarrow G$ by $(h,g) \mapsto hg$. Show that $g_1, g_2 \in G$ are in the same orbit of this action if and only if $g_1g_2^{-1}$ is an element of $H$.
   (b) Define a group action $H \times G \rightarrow G$ by $(h,g) \mapsto h \cdot g = gh^{-1}$. Show that
      (i) this is indeed a group action,
      (ii) $g_1, g_2 \in G$ are in the same orbit of this action if and only if $g_2^{-1}g_1$ is an element of $H$.

2. Let $T(E^2)$ be the set of all translations of the Euclidean plane. Show that $T(E^2)$ is a subgroup of $I(E^2)$.

3. Define a function $f : I(E^2) \rightarrow T(E^2)$ by $f(S) = T_{S(0)}$ (i.e., $f(S)$ is the translation by vector $S(0)$).
   (a) Show that $f$ is onto.
   (b) Show that $f(S_1) = f(S_2)$ if and only if $S_2^{-1}S_1$ is an element of $O(2)$, the group of Euclidean transformations that preserve the origin.

4. Let $f : I(E^2) \rightarrow T(E^2)$ be the function of Problem 3. Let $O(2)$ act on $I(E^2)$ as follows: $Q \cdot S = SQ^{-1}$. Let $I(E^2)/O(2)$ be the corresponding quotient space.
   (a) Show that $f$ maps each orbit of this action to a single point.
   (b) Define $\overline{f} : I(E^2)/O(2) \rightarrow T(E^2)$ by $\overline{f}(\text{orbit of } S) = f(S)(= S(0))$ and show that it is one-to-one. (Hint: Use the fact that $S_1$ and $S_2$ are in the same orbit if and only if $S_2^{-1}S_1$ is an element of $O(2)$ (Problem 1(a)(ii)), and (b) of the previous problem.)
   (c) Use (b) and Problem 3 to show that $\overline{f}$ is a bijection. This will show that $I(E^2)/O(2)$ can be identified with the group of translations.

5. Show that the function $\phi : \text{Stab}_{I(E^2)}(P) \rightarrow O(2)$, defined by $f(S) = T_{a}^{-1}ST_a$, where $a$ is the vector $\overrightarrow{OP}$, is an isomorphism.